Attitude Stability of Dual-Spin Spacecraft with Unsymmetrical Bodies

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This paper investigates the attitude stability of dual-spin spacecraft with unsymmetrical bodies. Of particular interest is the unstable attitude behavior that can result during the despin maneuver of the spacecraft's platform or rotor. The attitude stability of several representative spacecraft configurations is investigated. The results indicate that the likelihood of unstable attitude behavior decreases as the "spin-up" body becomes more symmetric and the "spin-down" body becomes less so. The change in the relative rotation rate between the spacecraft's platform and rotor is assumed to be slow compared to the precession rate. The attitude stability information presented evolves from the application of Hill's infinite determinant method, with the results confirmed by Floquet theory. Parameter selections for the Floquet study were guided by the results predicted by Hill's method. Further, the analysis results are compared to those of Tsuchiya, which are based on the method of averaging. The present work shows that the stability regions are rich in details which have not been identified by previous investigations.

I. Introduction

OVER the years, dual-spin satellites have performed successfully in spacecraft missions where the Earth-orbited body is required to maintain a direct line of sight with a specified location on the Earth's surface, such as a ground station. This design permits the Earth-fixed portion of the spacecraft containing the antennae (the platform) to rotate in synchronism with the Earth's rotation, while the spinning portion of the spacecraft (the rotor) provides the angular momentum necessary for the gyrostabilization of the spacecraft.

After separation of the spacecraft from its launch vehicle, both the platform and the rotor are spun up as a unit using thrusters. The spacecraft is then injected into synchronous orbit. Since the platform and rotor are rotating with identical angular velocities, this particular operational mode of the spacecraft is referred to as its "all-spun" condition. The platform must now be "despun" until it is pointing at the Earth for the spacecraft to be fully operational.

An important area of investigation pertaining to spacecraft performance is the identification of unstable attitude behavior. During unstable attitude behavior, the nutation angle between the spacecraft's spin axis and angular momentum vector grows. Unstable attitude behavior may be encountered during the process of despinning the platform (or rotor). This dynamic process requires the relative rotation rate between the platform and rotor to change. If the relative rotation rate changes slowly when compared to the spacecraft's precession rate, it may be assumed essentially constant. For this condition, information pertaining to the attitude stability of the spacecraft may be obtained. This assumption is inherent to the stability analysis results presented in this paper. Using these results, one may speculate as to the spacecraft's actual attitude behavior when the relative rotation rate is time varying.

Received Aug. 23, 1983; revision received Nov. 22, 1983.

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Much of the early work in the field of attitude stability of dual-spin spacecraft dealt with those particular configurations having at least one symmetric body.[1,2] Tsuchiya[3] addressed questions on the attitude stability of a class of dual-spin spacecraft in which both bodies were asymmetric and free from energy dissipation. Tsuchiya showed for this class of spacecraft that it was possible for unstable behavior of the spacecraft to result in its pure rotation mode.

The present work basically addresses the same problem considered by Tsuchiya. However, in this investigation the asymmetry of the platform and rotor may be arbitrarily assigned. In addition, the present investigation examines the relationships among the spacecraft's physical size, shape, and the analysis parameters defining the asymmetries of the platform and rotor. Further, limitations pertaining to these parameters which ensure the existence of a physically realizable spacecraft configuration are presented.

In Sect. II, the full nonlinear equations of motion for the idealized dual-spin spacecraft model are developed and linearized about the equilibrium solution that represents the desired attitude motion of the spacecraft. The resulting linearized variational equations having periodic coefficients are nondimensionalized by defining appropriate combinations of parameters.

Section III deals with methods of stability analysis. The first one discussed is Hill's method of infinite determinants. Hill's method is inherently approximate and yields analytical expressions for boundary curves which define regions of unstable attitude behavior. The second method discussed is Floquet theory. The application of Floquet theory yields numerical results from which attitude stability is determined. This method is exact except for approximations inherent in a numerical implementation. The primary disadvantage of Floquet analysis is that results are cast in numerical, rather than analytical, form. This requires extensive exploration of the parameter space via digital computer to discover general regions of stability. This difficulty may be partially alleviated by using the results obtained from Hill's method to identify regions of the parameter space to investigate in the Floquet analysis. In addition, the results obtained from the Floquet analysis serve to validate Hill's approximate analysis results.

In Sect. IV, attitude stability information is presented for a variety of spacecraft configurations. Selected results from Hill's approximate analysis are confirmed using Floquet theory. In addition, Tsuchiya's attitude stability information, which evolved from the method of averaging, is compared with the results obtained using Hill's method. Further, speculation as to the actual attitude behavior of the spacecraft is presented for the case when the relative rotation rate between the platform and rotor is time varying.

A summary of the findings from this investigation is presented in Sect. V.

II. Analytical Model

The idealized model defining the dual-spin spacecraft is shown in Fig. 1. The system includes two asymmetrical rigid bodies, B and B', constrained to relative rotation about the spacecraft spin axis, which is also a centroidal principal axis of inertia for both bodies. The relative angular displacement is θ and the relative angular rate is σ.

Let the mutually perpendicular axes xyz and $x'y'z'$, possessing unit vectors $\boldsymbol{xyz}$ and $\boldsymbol{x'y'z'}$, respectively, be defined for bodies B and B'. These axes have a common origin at the system's mass center, C. Let I_1, I_2, I_3, and I_1', I_2', I_3' represent the moments of inertia of bodies B and B' for axes xyz and $x'y'z'$, respectively. The xyz reference frame has angular velocity components $\omega_1, \omega_2, \omega_3$, and the $x'y'z'$ reference frame has components $\omega_1', \omega_2', \omega_3'$. In addition, a Newtonian reference frame XYZ is defined with its origin at C. In describing the motion of the spacecraft, body B has been chosen as the reference body with body B' the secondary body. Hence, the motion of B' will be transformed to the xyz coordinate system fixed in B.

The equations of motion for the analytical model are obtained by considering the time rate of change of the system's angular momentum in the Newtonian reference frame:

$$\frac{{}^{(N)}\mathrm{d}\boldsymbol{H}_T}{\mathrm{d}t} = \frac{{}^{(B)}\mathrm{d}\boldsymbol{H}_T}{\mathrm{d}t} + \boldsymbol{\omega}^{BN} \times \boldsymbol{H}_T \tag{1}$$

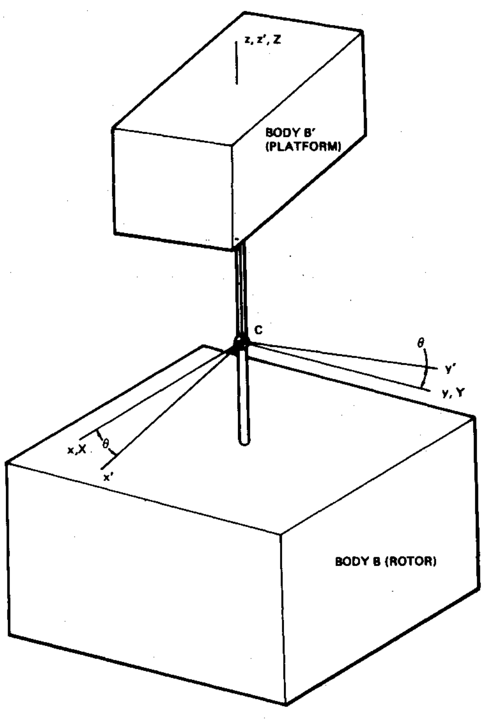

Fig. 1 Idealized model of dual-spin spacecraft.

where

$$\boldsymbol{\omega}^{BN} = \omega_1 \boldsymbol{x} + \omega_2 \boldsymbol{y} + \omega_3 \boldsymbol{z} \tag{2}$$

$$\begin{aligned} \boldsymbol{H}_T = \big[& I_1\omega_1 + I_1'\left(\omega_1\cos^2\theta + \omega_2\sin\theta\cos\theta\right) \\ & + I_2'\left(\omega_1\sin^2\theta - \omega_2\sin\theta\cos\theta\right)\big]\,\boldsymbol{x} \\ & + \big[I_2\omega_2 + I_1'\left(\omega_1\sin\theta\cos\theta + \omega_2\sin^2\theta\right) \\ & + I_2'\left(-\omega_1\sin\theta\cos\theta + \omega_2\cos^2\theta\right)\big]\,\boldsymbol{y} \\ & + \left[I_3\omega_3 + I_3'(\omega_3 + \sigma)\right]\boldsymbol{z} \end{aligned} \tag{3}$$

$$\sigma = \dot\theta = \text{const} \tag{4}$$

Since the spacecraft is in a force-free environment, no external torques act, and Eq. (1) may be equated to zero. Substituting Eqs. (2) and (3) into Eq. (1) and carrying out the required vector differentiation and cross-product operations, we obtain the following equations of motion:

$$\begin{aligned} & \left[I_1 + I_1' - (I_1' - I_2')\sin^2\theta\right]\dot\omega_1 + \left[(I_1' - I_2')\sin\theta\cos\theta\right]\dot\omega_2 \\ & + \left[(I_2' - I_1')\sigma\sin 2\theta\right]\omega_1 + \left[(I_1' - I_2')\sigma\cos 2\theta + I_3'\sigma\right]\omega_2 \\ & + \left[I_3 + I_3' - I_2 - I_2' - (I_1' - I_2')\sin^2\theta\right]\omega_2\omega_3 \\ & + \left[(I_2' - I_1')\sin\theta\cos\theta\right]\omega_1\omega_3 = 0 \end{aligned} \tag{5}$$

$$\begin{aligned} & \left[(I_1' - I_2')\sin\theta\cos\theta\right]\dot\omega_1 + \left[I_2 + I_2' + (I_1' - I_2')\sin^2\theta\right]\dot\omega_2 \\ & + \left[(I_1' + I_2')\sigma\cos 2\theta - I_3'\sigma\right]\omega_1 + \left[(I_1' - I_2')\sigma\sin 2\theta\right]\omega_2 \\ & + \left[(I_1' - I_2')\sin\theta\cos\theta\right]\omega_2\omega_3 \\ & + \left[I_1 + I_1' - I_3 - I_3' - (I_1' - I_2')\sin^2\theta\right]\omega_1\omega_3 = 0 \end{aligned} \tag{6}$$

$$\begin{aligned} & \left[I_3 + I_3'\right]\dot\omega_3 + \left[(I_1' - I_2')\sin\theta\cos\theta\right]\omega_1^2 \\ & + \left[(I_2' - I_1')\sin\theta\cos\theta\right]\omega_2^2 \\ & + \left[I_2 - I_1 + (I_2' - I_1')\cos 2\theta\right]\omega_1\omega_2 = 0 \end{aligned} \tag{7}$$

For the set of nonlinear equations of motion, Eqs. (5-7), the equilibrium solution representing the desired attitude motion of the spacecraft requires that the z axis remain parallel to the Z inertial axis, while bodies B and B' rotate, respectively, with constant angular rates Ω and $\Omega + \sigma$. In mathematical terms, this motion corresponds to the solution

$$\omega_1 = \omega_2 = 0 \qquad \omega_3 = \Omega \tag{8}$$

The parameters ω_1, ω_2, and ω_3 are the angular velocity components of B in the Newtonian reference frame XYZ. The stability of this motion is of particular interest.

In order to ensure asymptotic stability of this solution, where ω_1 and ω_2 approach zero with time, the initial values for ω_1, ω_2, and $(\omega_3 - \Omega)$ are required to be sufficiently small but nonzero.

Determination of the stability of Eq. (8) may be obtained from examining the set of equations which results when Eqs.

(5-7) are linearized in ω_1 and ω_2, but not in ω_3:

$$[I_1 + I_1' + \tfrac{1}{2}(I_1' - I_2')(\cos 2\theta - 1)]\,\dot{\omega}_1 + [\tfrac{1}{2}(I_1' - I_2')\sin 2\theta]\,\dot{\omega}_2 - [(I_1' - I_2')(\sigma + \tfrac{1}{2}\omega_3)\sin 2\theta]\,\omega_1 + \{\omega_3(I_3 + I_3' - I_2 - I_2') + I_3'\sigma + (I_1' - I_2')[\sigma\cos 2\theta + \tfrac{1}{2}\omega_3(\cos 2\theta - 1)]\}\,\omega_2 = 0 \tag{9}$$

$$[\tfrac{1}{2}(I_1' - I_2')\sin 2\theta]\,\dot{\omega}_1 + [(I_2 + I_2') + \tfrac{1}{2}(I_1' - I_2')(1 - \cos 2\theta)]\,\dot{\omega}_2 + \{\omega_3(I_1 + I_1' - I_3 - I_3') - I_3'\sigma + (I_1' - I_2')[\sigma\cos 2\theta + \tfrac{1}{2}\omega_3(\cos 2\theta - 1)]\}\,\omega_1 + [(I_1' - I_2')(\sigma + \tfrac{1}{2}\omega_3)\sin 2\theta]\,\omega_2 = 0 \tag{10}$$

$$[I_3 + I_3']\,\dot{\omega}_3 = 0 \tag{11}$$

Equation (11) possesses the solution

$$\omega_3 = \Omega \tag{12}$$

This result indicates that ω_3 remains essentially constant, even in the presence of small transverse angular motions. Substituting Ω for ω_3 in Eqs. (9) and (10) yields the variational equations which govern the nutational motion of the system.

Investigation into the attitude stability of the spacecraft may be facilitated by the introduction of the following dimensionless quantities into the equations of motion:

$J = \dfrac{I_1 + I_1' + I_2 + I_2'}{2}$,	Transverse inertia parameter (FLT^2)	(13)
$h = I_3\Omega + I_3'(\Omega + \sigma)$,	Spin axis angular momentum parameter (FLT)	(14)
$\epsilon = \dfrac{I_1 - I_2}{2J} \overset{\Delta}{\geq} 0$,	Asymmetry parameter for body B (nondimensional)	(15)
$\bar{\epsilon} = \dfrac{I_1' - I_2'}{2J} \overset{\Delta}{\geq} 0$,	Asymmetry parameter for body B' (nondimensional)	(16)
$r = \dfrac{J\Omega}{h}$,	Normalized spin rate parameter for body B (nondimensional)	(17)
$\bar{r} = \dfrac{J(\Omega + \sigma)}{h}$,	Normalized spin rate parameter for body B' (nondimensional)	(18)
$\tau = \dfrac{ht}{J};\ \dfrac{\mathrm{d}}{\mathrm{d}\tau}(\ ) \triangleq (\ )'$,	Normalized time parameter (nondimensional)	(19)
$\alpha = (\bar{r} - r)\tau$,	Circular frequency parameter (nondimensional)	(20)

In the preceding list, J and h are the only quantities which are not dimensionless. In addition, note that the transverse axes defining each body's inertia properties may be labeled such that ϵ and $\bar{\epsilon}$ are always positive. Upon substituting these quantities, and Ω for ω_3, into Eqs. (9) and (10), the variational equations may be written in the following form:

$$[M_1]\{\omega'\} = [M_2]\{\omega\} \tag{21}$$

where

$$\{\omega\} = [\omega_1 \quad \omega_2]^T \tag{22}$$

$$[M_1] = \begin{bmatrix} 1 + \epsilon + \bar{\epsilon}\cos 2\alpha & \bar{\epsilon}\sin 2\alpha \\ \bar{\epsilon}\sin 2\alpha & 1 - \epsilon - \bar{\epsilon}\cos 2\alpha \end{bmatrix} \tag{23}$$

$$[M_2] = \begin{bmatrix} \bar{\epsilon}(2\bar{r} - r)\sin 2\alpha & -1 - r(\epsilon - 1) - \bar{\epsilon}(2\bar{r} - r)\cos 2\alpha \\ 1 - r(1 + \epsilon) - \bar{\epsilon}(2\bar{r} - r)\cos 2\alpha & -\bar{\epsilon}(2\bar{r} - r)\sin 2\alpha \end{bmatrix} \tag{24}$$

The coefficients appearing in variational Eq. (21) are periodic with period $\pi/[\bar{r} - r]$. Information concerning the stability of the zero solution of these equations presently under consideration can be obtained using Floquet theory (exact) or Hill's method of infinite determinants (approximate).

The introduction of the nondimensional parameters simplifies the variational equations considerably. Insight as to the physical configuration of the spacecraft may be obtained by considering a parameter plane defined by the quantities ϵ and $\bar{\epsilon}$.

Substituting Eq. (13) into Eqs. (15) and (16) results in the expression

$$\epsilon + \bar{\epsilon} = 1 - \frac{2(I_2 + I_2')}{I_1 + I_1' + I_2 + I_2'} \tag{25}$$

Suppose I_2 and I_2' are allowed to approach (but not equal) zero. Accordingly, the platform and rotor each assume the shape of a long slender rod. Equation (25) becomes

$$\epsilon + \bar{\epsilon} \approx 1 \tag{26}$$

Next, consider the spacecraft configurations which result when $\epsilon = \bar{\epsilon}$. Along this line, both the platform and rotor are asymmetric to the same degree. Also, since the platform could have been chosen as the reference body for the analysis, whereby the roles of ϵ and $\bar{\epsilon}$ would be interchanged, the line $\epsilon = \bar{\epsilon}$ becomes an axis of symmetry. Therefore, the region of the $(\epsilon, \bar{\epsilon})$ parameter plane isolated for the stability analysis is bounded by Eq. (26) and the line $\epsilon = \bar{\epsilon}$. This region is shown in Fig. 2.

With respect to Fig. 2, first observe the spacecraft configurations defined along Path 1. Along Path 1, $\epsilon = \bar{\epsilon}$. This stipulation requires the platform and rotor to grow asymmetric to the same degree. At the point $(\epsilon, \bar{\epsilon}) = (0.5, 0.5)$, Eq. (26) must also

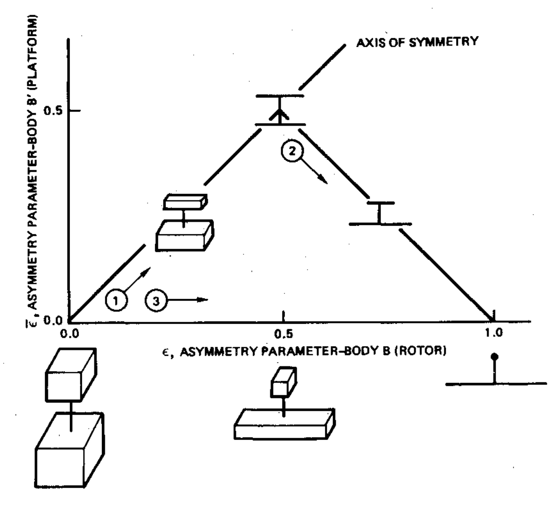

Fig. 2 Physical interpretation of the nondimensional parameters ϵ and $\bar{\epsilon}$.

be satisfied. Accordingly, the platform and rotor each have the shape of a long slender rod. Continuing along Path 2 results in the platform growing symmetric once again while the rotor continues to grow asymmetrically. At the point $(\epsilon, \bar{\epsilon}) = (1,0)$, the platform is required to be symmetric in conjunction with I_2' approaching zero. This requires the platform to be shaped as an infinitesimal cube or point mass, while the rotor's shape is that of a long, slender rod. Next, consider Path 3. Along Path 3, $\bar{\epsilon} = 0$, which requires the platform to remain symmetric while the rotor grows asymmetrically.

At this point it is interesting to comment on the restrictions imposed by Tsuchiya[3] in his stability analysis regarding the physical configuration of the spacecraft. Tsuchiya defines the moments of inertia of the platform and rotor as follows:

$$I_1 = I_1' = I + \Delta \tag{27}$$

$$I_2 = I_2' = I - \Delta \tag{28}$$

$$I_3 = I_3' \tag{29}$$

$$\xi = \frac{\Delta}{2I} \tag{30}$$

The parameter ξ is a measure of the asymmetry of the spacecraft configuration. Subtracting Eq. (28) from Eq. (27) results in the relation

$$\Delta = \frac{I_1 - I_2}{2} = \frac{I_1' - I_2'}{2} \tag{31}$$

Combining the definition J from Eq. (13) with the definitions for I_1, I_1', I_2, and I_2' given in Eqs. (27) and (28) results in the expression

$$J = 2I \tag{32}$$

In addition, combining Eqs. (30-32), we obtain the relation

$$\xi = \frac{I_1 - I_2}{2J} = \frac{I_1' - I_2'}{2J} \tag{33}$$

This relation may be simplified by recalling the definitions for ϵ and $\bar{\epsilon}$ given in Eqs. (15) and (16). Hence,

$$\xi = \epsilon = \bar{\epsilon} \tag{34}$$

Therefore, Tsuchiya's stability analysis of dual-spin spacecraft is limited to spacecraft configurations having a platform and rotor which are identically shaped and lie on Path 1.

Considering the physical significance of the parameters ϵ and $\bar{\epsilon}$, we assume that they are known time-invariant quantities in the variational equations (21). Once values have been specified for ϵ and $\bar{\epsilon}$, one can focus on the parameter plane defined by the quantities r and $\bar{r}$. The $(r, \bar{r})$ plane is convenient for studying attitude behavior during the despin maneuver.

With the introduction of the nondimensional parameters there exists the possibility that for certain magnitudes of r and $\bar{r}$, the spacecraft configuration may not be physically realizable. From elementary mechanics, the sum of any two moments of inertia cannot be less than the third. Hence,

$$I_1 + I_2 \geq I_3 \tag{35}$$

$$I_1' + I_2' \geq I_3' \tag{36}$$

Adding Eqs. (35) and (36), we obtain

$$I_1 + I_2 + I_1' + I_2' \geq I_3 + I_3' \tag{37}$$

Recalling the definition for J from Eq. (13), Eq. (37) may be written in the following form:

$$J \geq \frac{I_3 + I_3'}{2} \tag{38}$$

In addition, on combining Eqs. (14), (17), and (18) we have

$$J = I_3 r + I_3' \bar{r} \tag{39}$$

Hence, combining Eqs. (38) and (39), we obtain

$$I_3\left(r - \tfrac{1}{2}\right) + I_3'\left(\bar{r} - \tfrac{1}{2}\right) \geq 0 \tag{40}$$

Therefore, for the spacecraft configuration to be physically realizable, Eq. (40) must be satisfied.

Suppose now the following parameters are defined:

$$\rho = r - \tfrac{1}{2} \tag{41}$$

$$\bar{\rho} = \bar{r} - \tfrac{1}{2} \tag{42}$$

$$\mu = \frac{I_3}{I_3'} \tag{43}$$

Substituting Eqs. (41-43) into Eq. (40) results in

$$\mu\rho + \bar{\rho} \geq 0 \tag{44}$$

Equation (44) must also be satisfied to ensure the existence of a physically realizable spacecraft configuration. Note that from physical considerations μ is always a positive quantity such that $0 < \mu < \infty$. Consider now a parameter plane defined by $(\rho, \bar{\rho})$ and the following arbitrary line lying in this plane:

$$\bar{\rho} = -\mu\rho + b \tag{45}$$

In this expression, b is the $\bar{\rho}$ axis intercept, and the negative sign results from Eq. (44) with equality holding.

Assume next that $0 < b < \infty$. Equation (44) is satisfied for all b such that $0 < b < \infty$ and all μ such that $0 < \mu < \infty$. Therefore, all lines corresponding to a physically realizable spacecraft configuration must intersect the positive $\bar{\rho}$ axis.

Further, as a check of the previous results, suppose that $0 > b > -\infty$. Equation (44) is not satisfied for any b such that $0 > b > -\infty$. Hence, the spacecraft is not physically realizable for any line intersecting the negative $\bar{\rho}$ axis.

From these arguments, a region of the $(\rho, \bar{\rho})$ parameter plane evolves which corresponds to spacecraft configurations that are not physically realizable. This region in the $(r, \bar{r})$ parameter plane is shown in Fig. 3.

III. Methods of Analysis

Hill's infinite determinant method[4] was developed to study the stability of dynamical systems which are represented by linear, homogeneous, ordinary differential equations with periodic coefficients.

Consider the matrix variational equation of motion, Eq. (21). Information concerning the stability of the system may be obtained from determining the characteristic multipliers of the system's fundamental matrix. For the second-order system presently under consideration, the characteristic equation may be represented by

$$\lambda^2 - 2A\lambda + 1 = 0 \tag{46}$$

In this expression, A is a constant determined from the parameters r, $\bar{r}$, ϵ, and $\bar{\epsilon}$. Solving Eq. (46), we obtain

$$\left.\begin{matrix}\lambda_1 \\ \lambda_2\end{matrix}\right\} = A \pm \sqrt{A^2 - 1} \tag{47}$$

For $A^2 > 1$, the characteristic multipliers λ_1 and λ_2 are real

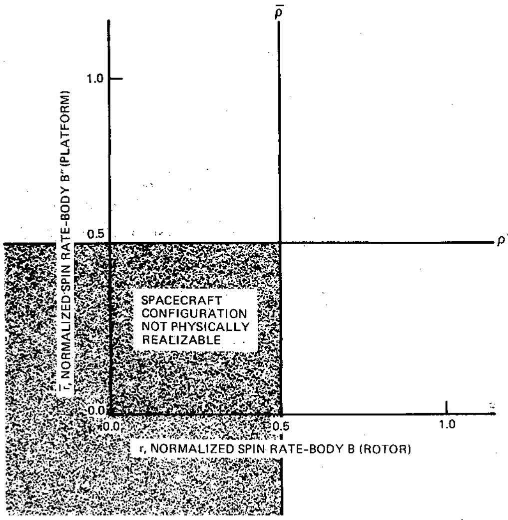

Fig. 3 Physically realizable regions of the $(r,\bar{r})$ parameter plane.

such that $0<\lambda_2<1<\lambda_1$ or $\lambda_2<-1<\lambda_1<0$. The solution of the system in this case is unbounded. For $A^2<1$, the characteristic multipliers λ_1 and λ_2 are complex conjugates both having absolute values equal to 1. In this case, nothing may be proven rigorously regarding the boundedness of the solution, although a bounded solution is suggested. When $A^2=1$, the characteristic multipliers assume the values $\lambda_1=\lambda_2=1$ or $\lambda_1=\lambda_2=-1$, and in either of these cases, Eq. (21) has at least one periodic solution. Hence, those particular solutions which correspond to periodic solutions $(A^2=1)$ represent dividing lines between bounded $(A^2<1)$ and unbounded $(A^2>1)$ system behavior. In particular, when one of the characteristic multipliers of the system is equal to 1 $(A^2=1)$, there exists a periodic solution with period $T=\pi/|\bar{r}-r|$ and $\omega(\tau)=\omega(\tau+T)$. Furthermore, when one of the characteristic multipliers is equal to -1 $(A^2=1)$, a second periodic solution exists with period $2T=2\pi/|\bar{r}-r|$ and $\omega(\tau)=-\omega(\tau+T)$. This latter requirement is not noted in Ref. 4. A pictorial representation of these results is shown in Fig. 4. Therefore, the periodic solutions appear in the $(r,\bar{r})$ parameter plane for a given $(\epsilon,\bar{\epsilon})$ in the form of boundary curves separating regions of bounded (stable) and unbounded (unstable) system behavior.

This discussion leads to the following conclusions regarding the stability of the system's variational equations. If the periodic motion at both boundary curves of the region in the parameter plane possesses the same period T or $2T$, then the enclosed region is characterized by unbounded motion. If the periodic motion at the boundary curve of a region in the parameter plane possesses period T $(2T)$ and at the other boundary curve of the region possesses period $2T$ (T), then the motion in that region is bounded.[4]

Since the motion is periodic along the boundary curves separating regions of stable and unstable system behavior, the solution of the system's equations (21) can be approximated by the Fourier series

$$\omega(\tau)=\sum_{q=0}^{\infty}\left[a_q\cos(q\alpha)+b_q\sin(q\alpha)\right] \tag{48}$$

Recall from Eq. (20) that $\alpha=(\bar{r}-r)\tau$. This Fourier series has an infinite number of terms. In order to obtain the stability boundaries, the series must be truncated. The accuracy of the boundaries, and consequently of Hill's method in general, depends on the number of terms retained.

Consider first the period $2T$ solution. Since $\omega(\tau)=-\omega(\tau+T)$, the even order harmonics given by $q=0,2,4$, etc., are omitted from Eq. (48). Hence, the stability boundaries corresponding to the period $2T$ solution may be represented by the series

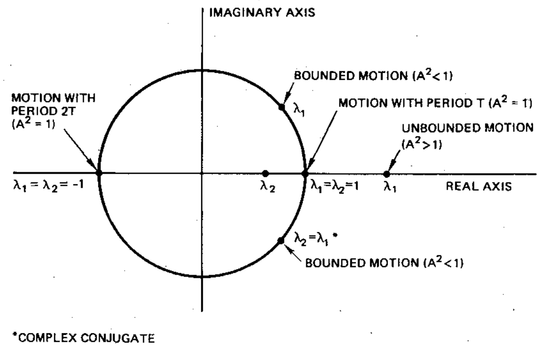

Fig. 4 Complex plane representation of characteristic multipliers.

$$\omega(\tau)=\sum_{q=1}^{\infty}\left\{a_q\cos[(2q-1)\alpha]+b_q\sin[(2q-1)\alpha]\right\} \tag{49}$$

Consider next the period T solution. Since $\omega(\tau)=\omega(\tau+T)$, the odd order harmonics given by $q=1,3,5$, etc., are omitted from Eq. (48). Hence, the stability boundaries corresponding to the period T solution may be represented by the series

$$\omega(\tau)=\sum_{q=0}^{\infty}\left[a_q\cos(2q\alpha)+b_q\sin(2q\alpha)\right] \tag{50}$$

For the present work, the period $2T$ and period T Fourier series solutions given by Eqs. (49) and (50), respectively, are truncated for $q\geq 2$. We will find that even though this truncation is quite severe, the results are surprisingly good.

Returning now to the period $2T$ solution, it is found that the stability boundaries corresponding to this solution may be obtained from Eq. (49). Truncating this series for $q\geq 2$ and applying Hill's infinite determinant methodology results in the following equations for the boundary curves:

$$[\epsilon^2-\bar{\epsilon}^2-1]\bar{r}^2+[2(1-\epsilon^2-\bar{\epsilon})r-\bar{\epsilon}]\bar{r}+[1-2r]=0 \tag{51}$$

$$[-\epsilon^2-\bar{\epsilon}^2+1]\bar{r}^2+[2(-1+\epsilon^2+\bar{\epsilon})r-\bar{\epsilon}]\bar{r}+[-1+2r]=0 \tag{52}$$

Expressions for the stability boundaries corresponding to the period T solution may be derived in a similar fashion from Eq. (50). Truncating this series for $q\geq 2$ and applying Hill's method results in the following equations:

$$\begin{aligned}&[4(\bar{\epsilon}^2-\epsilon^3+\epsilon^2+\epsilon-1)r-4\epsilon^2+4]\bar{r}^2\\&+[-8(\bar{\epsilon}^2-\epsilon^3+\epsilon^2+\epsilon-1)r^2+(2\bar{\epsilon}^2+8\epsilon^3-8)r]\bar{r}\\&+[3(\bar{\epsilon}^2-\epsilon^3+\epsilon^2+\epsilon-1)r^3+(-\bar{\epsilon}^2-3\epsilon^2+2\epsilon+1)r^2\\&+(3-\epsilon)r-1]=0\end{aligned} \tag{53}$$

$$\begin{aligned}&[-4(\bar{\epsilon}^2+\epsilon^3+\epsilon^2-\epsilon-1)r+4\epsilon^2-4]\bar{r}^2\\&+[8(\bar{\epsilon}^2+\epsilon^3+\epsilon^2-\epsilon-1)r^2+(-2\bar{\epsilon}^2-8\epsilon^3+8)r]\bar{r}\\&+[-3(\bar{\epsilon}^2+\epsilon^3+\epsilon^2-\epsilon-1)r^3\\&+(\bar{\epsilon}^2+3\epsilon^2+2\epsilon-1)r^2-(\epsilon+3)r+1]=0\end{aligned} \tag{54}$$

Hence, given numerical values for ϵ and $\bar{\epsilon}$, the stability boundaries may be generated for the $(r,\bar{r})$ parameter plane by solving Eqs. (51-54) for r and $\bar{r}$. Consequently, the regions corresponding to stable and unstable attitude behavior of the spacecraft may be identified.

On considering the theory of Floquet, recall the matrix variational equation of motion, Eq. (21), and note once again that the coefficient matrices are periodic with period $T=\pi/|\bar{r}-r|$. Let $[\phi(\tau,\tau_0)]$ represent the fundamental matrix for the system. In accordance with Floquet theory, the asymptotic behavior of solutions to this equation depends on the values for the characteristic multipliers at time $\tau=T$. The fundamental matrix $[\phi(\tau,\tau_0)]$ is defined by the differential equation

$$[M_1][\dot{\phi}]=[M_2][\phi] \tag{55}$$

and the initial condition

$$[\phi(0)]=[I] \tag{56}$$

The matrix $[I]$ is a 2×2 identity matrix.

Any solution $\{y(\tau)\}$, which represents a small initial disturbance of the system from its nominal equilibrium position, may be expressed in terms of $[\phi(\tau,\tau_0)]$ and the initial condition column matrix $\{\omega(\tau_0)\}$ in the following manner:

$$\{y(\tau)\}=[\phi(\tau,\tau_0)]\{\omega(\tau_0)\} \tag{57}$$

$$\{y(\tau)\}=\begin{Bmatrix} y_1(\tau) \\ y_2(\tau) \end{Bmatrix} \tag{58}$$

$$\{\omega(\tau_0)\}=\begin{Bmatrix} \omega_1(\tau_0) \\ \dot{\omega}_2(\tau_0) \end{Bmatrix} \tag{59}$$

It has been shown previously by Coddington and Levison[5] that if $|\lambda_i|<1$ for $i=1,2$, then $\{y(\tau)\}$ will go to zero as $\tau\rightarrow\infty$. If $|\lambda_i|\leq 1$, one can prove nothing rigorously concerning the stability of such systems; however, stability is suggested. If either λ_i satisfies $|\lambda_i|>1$, then the null solution will be unstable. For the undamped system at hand, we will never find λ_i such that $|\lambda_i|<1$. Hence, we concern ourselves with finding conditions such that $|\lambda_i|>1$. Therefore, we are only defining conditions for instability.

In the present analysis, a digital computer was used to determine the fundamental matrix $[\phi(T,0)]$. The procedure used to study the attitude behavior of the system is outlined as follows:

1) Assign numerical values to the parameters $\epsilon, \bar{\epsilon}$ and $r, \bar{r}$.

2) Numerically integrate Eq. (21) from $\tau=0$ to $\tau=T$ using the initial condition $[\phi(0)]=[I]$. Note that this requires determination of the inverse of $[M_1]$ which, because of its physical origin corresponding to the system's inertia, always exists.

3) Compute the characteristic multipliers of $[\phi(T,0)]$ and stability information from their locations in the complex plane.

4) Repeat steps 2 and 3 with new values of $r, \bar{r}$, but the same values of $\epsilon, \bar{\epsilon}$, until the entire parameter space of interest is explored.

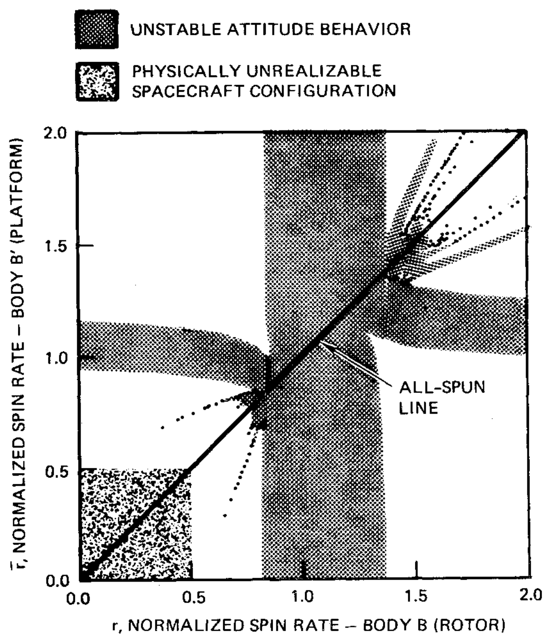

Fig. 6 Stability analysis results using Floquet theory for $\epsilon=0.25$, $\bar{\epsilon}=0.10$.

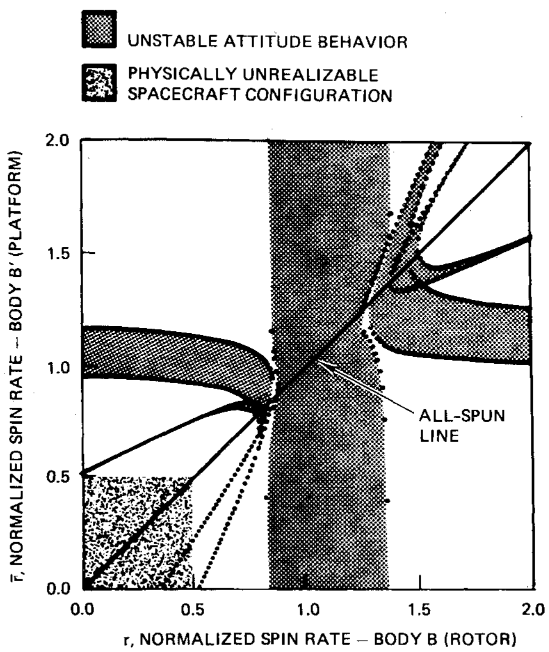

Fig. 5 Stability analysis results using Hill's method for $\epsilon=0.25$, $\bar{\epsilon}=0.10$.

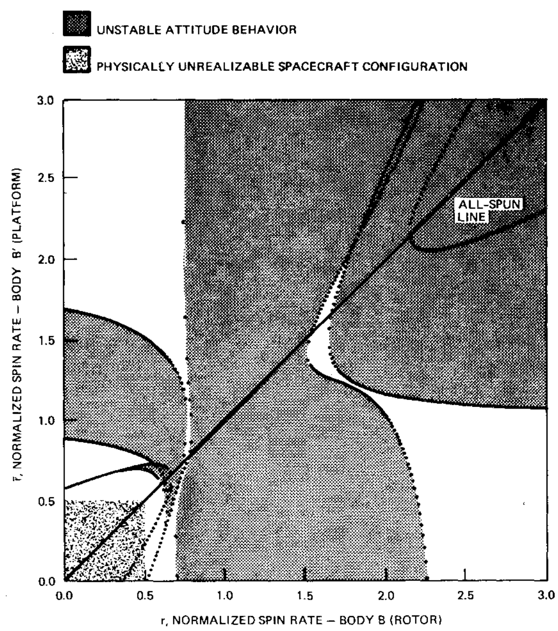

Fig. 7 Stability analysis results using Hill's method for $\epsilon=0.50$, $\bar{\epsilon}=0.25$.

IV. Stability Analysis

When Hill's infinite determinant method results are applied to a spacecraft configuration having mass properties such that $(\epsilon, \bar{\epsilon}) = (0.25, 0.10)$, the stability regions displayed in Fig. 5 are obtained. The application of Floquet theory with the same parameter values leads to the results shown in Fig. 6. Similarly, when the results of Hill's method are applied to a spacecraft configuration having $(\epsilon, \bar{\epsilon}) = (0.50, 0.25)$, the stability regions displayed in Fig. 7 result, and the corresponding stability regions predicted from the application of Floquet theory are shown in Fig. 8.

When the results shown in Figs. 5-8 are compared, it is clear that the correlation is excellent among all primary regions of unstable attitude behavior as predicted by the two methods of analysis for both spacecraft configurations. However, it appears that the higher-order parametric resonance lines appearing in the Floquet analysis results are not present in the results obtained using Hill's method. This may be attributed to the limited number of terms retained in the Fourier series representation of the period T and period $2T$ solutions to the variational equations of motion. Also, slender areas where unstable attitude behavior has not been identified appear in the results obtained from Hill's method near the sets of points $(r, \bar{r}) = [(0.9, 1.0), (1.3, 1.2)]$ and $(r, \bar{r}) = [1.6, 1.5]$ for the spacecraft configurations $(\epsilon, \bar{\epsilon}) = (0.25, 0.10)$ and $(\epsilon, \bar{\epsilon}) = (0.50, 0.25)$, respectively. These areas do not appear in the Floquet analysis results. However, for the $(\epsilon, \bar{\epsilon}) = (0.50, 0.25)$ configuration, there is partial agreement between the analysis methods in the slender area predicted near the point $(r, \bar{r}) = (0.7, 1.2)$. Interestingly, this small area is bounded completely by unstable regions in the Floquet analysis results. Since the slender areas are not detected by the more rigorous Floquet analysis (with the one exception), it is felt that they are actually part of the neighboring regions of instability even though Hill's method indicates otherwise. This discrepancy is assumed to result from the previously mentioned approximations inherent to Hill's method. Numerical integration of the full nonlinear equations would be required to address this issue. The attitude stability of spacecraft configurations having mass properties such that $(\epsilon, \bar{\epsilon}) = (0.25, 0.25)$, $(0.50, 0.10)$, $(0.50, 0.50)$, $(0.75, 0.10)$, $(0.75, 0.25)$ has been investigated using Hill's method.[6]

The attitude stability of the spacecraft may also be investigated using the method of averaging. This approach was implemented by Tsuchiya in his study.[3] Tsuchiya concluded that if the spin angular velocity of either of the two bodies is near the angular velocity of nutational body motion, it is possible for the spacecraft to be unstable in its pure rotation mode. The criterion for unstable attitude behavior established by Tsuchiya may be cast into the following form using combinations of parameters given in Eqs. (13-19):

$$\frac{1}{1+\epsilon} < r < \frac{1}{1-\epsilon} \tag{60}$$

$$\frac{1}{1+\bar{\epsilon}} < \bar{r} < \frac{1}{1-\bar{\epsilon}} \tag{61}$$

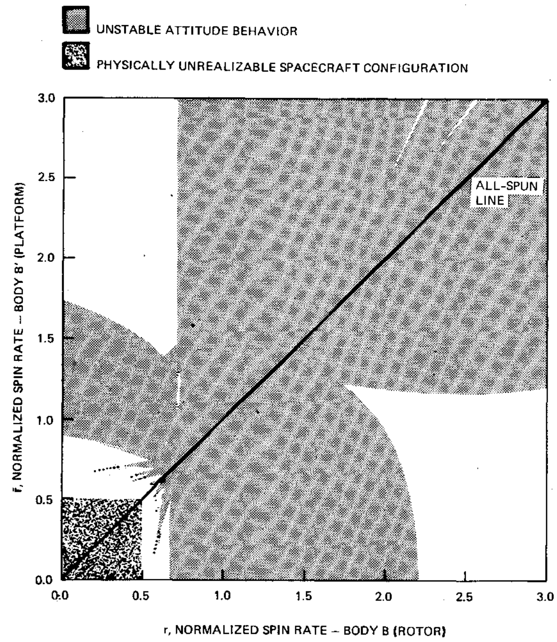

Fig. 8 Stability analysis results using Floquet theory for $\epsilon = 0.50$, $\bar{\epsilon} = 0.25$.

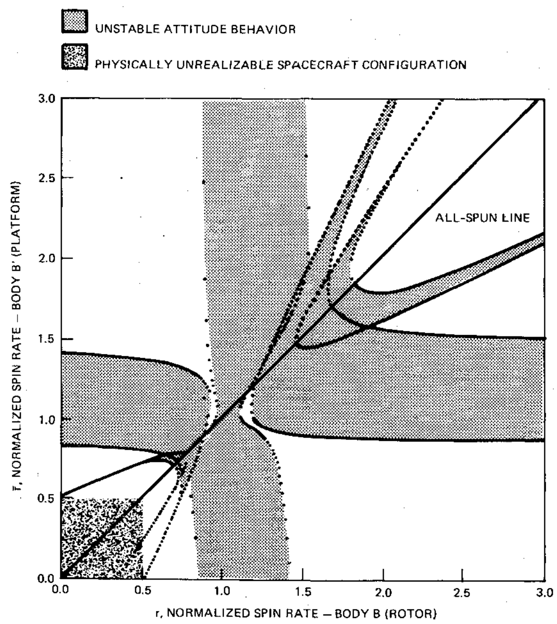

Fig. 9 Stability analysis results using Hill's method for $\epsilon = 0.25$, $\bar{\epsilon} = 0.25$.

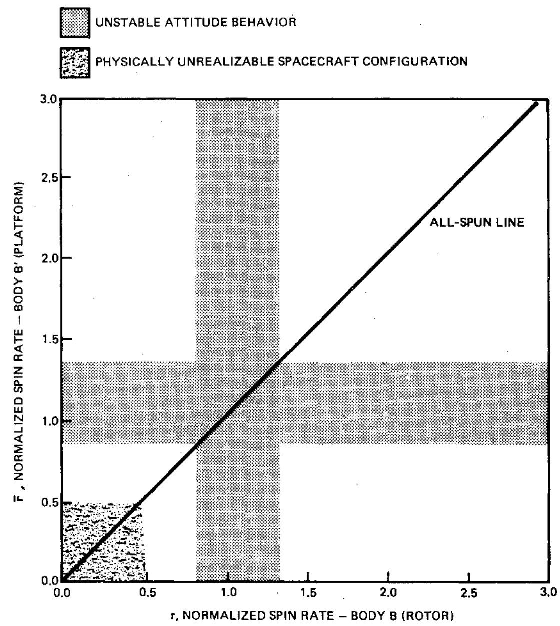

Fig. 10 Tsuchiya's stability analysis results using the method of averaging for $\epsilon = 0.25$, $\bar{\epsilon} = 0.25$.

The attitude stability of a spacecraft configuration having mass properties such that $(\epsilon, \bar{\epsilon}) = (0.25, 0.25)$ has previously been investigated using Hill's infinite determinant method with the results shown in Fig. 9. The same spacecraft configuration may be analyzed via the application of stability criteria given by Eqs. (60) and (61). The resulting regions of unstable attitude behavior are shown in Fig. 10.

Comparing the stability results shown in Figs. 9 and 10, it is clear that the stability criterion predicted by Tsuchiya is only an approximation of the results obtained using Hill's method. This discrepancy occurs because of the level of approximation used by Tsuchiya in the application of the averaging method. If higher-order averaging were employed, one would expect the results to be in better agreement.

It also is interesting to compare Tsuchiya's stability criterion given in Eqs. (60) and (61) to the analytic stability criterion of the present investigation. Consider first a spacecraft configuration where the platform is asymmetric ($\bar{\epsilon} > 0$) and the rotor is symmetric ($\epsilon = 0$). For this configuration, the stability criterion evolving from the present work is identical to that given in Eq. (61). Alternately, consider the configuration where the platform is symmetric ($\bar{\epsilon} = 0$) and the rotor is asymmetric ($\epsilon > 0$). For this configuration, the stability criterion is identical to that given in Eq. (60). Hence, we see that Tsuchiya's stability criterion corresponds to a superposition of results in which one of the spacecraft's bodies is symmetric and the other is asymmetric.

Considering now the spacecraft's actual despin maneuver, note that the relative rotation between the platform and rotor $(\bar{r} - r)$ is time varying. Therefore, strictly speaking, the stability analysis results presented do not apply to the despin maneuver. Thus, one can only hope the results are meaningful when the change in $(\bar{r} - r)$ is slow compared to the spacecraft's precession rate. In this light, it is possible that for an appropriate choice in magnitude for the relative rotation angular acceleration, the unstable attitude behavior will be bounded. That is, the nutation angle will grow during the period when the despin maneuver of the spacecraft traverses instability regions. However, once crossing of the instability region is completed, the nutation angle will cease to grow and possess a finite value. Hence, the despin maneuver of the spacecraft may be successfully completed and a flat-spin condition avoided. Further investigation is required in this area to ensure the existence of and bounds corresponding to such a condition.

V. Summary

The attitude stability of two dual-spin spacecraft configurations has been investigated using Hill's method of infinite determinants with the results confirmed through the application of Floquet theory. Since Floquet theory does not yield analytic stability criteria, it has the disadvantage of requiring extensive exploration of the parameter plane with a digital computer to obtain the desired stability information for each spacecraft configuration. Therefore, Hill's infinite determinant method was employed to gather analytic stability criteria, which permitted a more judicious choice to be made for the analysis parameters used in the Floquet study.

Assuming that the change in the relative rotation angular velocity between the spacecraft's platform and rotor is slow compared to the precession rate, one may deduce possible trends in unstable attitude behavior encountered during the despin maneuver from the results presented and those given in Ref. 6. Hence, using this assumption, the likelihood of unstable attitude behavior decreases as the "spin-up" body becomes more symmetric and the "spin-down" body becomes less so. The effect of the magnitude of this relative rotation angular acceleration on the nutation angle growth encountered as the spacecraft traverses instability regions is a topic for further investigation. It is conceivable that for an appropriate choice in magnitude, the nutation angle growth exhibited may be small, allowing the spacecraft to be completely despun, thus avoiding a flat-spin condition.

The stability analysis of dual-spin spacecraft performed previously by Tsuchiya was discovered to be limited in scope. As a result of the analysis parameters chosen, Tsuchiya's analytic stability criterion may only be applied to spacecraft configurations having a platform and rotor which are identically shaped. Further, the level of approximation employed by Tsuchiya in the application of the averaging method results in attitude stability information which is highly approximate when compared with the results presented in this investigation using Hill's method of infinite determinants.

Acknowledgments

The work described in this paper was performed at the University of California, Los Angeles, California, and was used in partial fulfillment of the requirements for the degree of Master of Science in Engineering.

References

[1] Likins, P.W., "Attitude Stability Criteria for Dual-Spin Spacecraft," *Journal of Spacecraft and Rockets*, Vol. 4, Dec. 1967, pp. 1638-1643.

[2] Mingori, D.L., "Effects of Energy Dissipation on the Attitude Stability of Dual-Spin Satellites," *AIAA Journal*, Vol. 7, Jan. 1969, pp. 20-27.

[3] Tsuchiya, K. and Saito, H., "Effect of Asymmetry of Bodies on the Attitude Stability of a Dual Spin Spacecraft," *Proceedings of the 10th International Symposium on Space Technology and Science*, Tokyo, Japan, Sept. 3-8, 1973, pp. 1071-1078.

[4] Meirovitch, L., *Methods of Analytical Dynamics*, McGraw-Hill, New York, N.Y., 1970, pp. 264-270 and 280-288.

[5] Coddington, E.A. and Levinson, N., *Theory of Ordinary Differential Equations*, McGraw-Hill, New York, N.Y., 1955, pp. 78-81.

[6] Lukich, M.S., "The Attitude Stability of Dual-Spin Spacecraft with Asymmetrical Bodies," Master's Thesis, University of California, Los Angeles, Calif., Dec. 1982.